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Book Review: *Turbulence, Coherent Structures, Dynamical Systems, and Symmetry*

Turbulence, Coherent Structures, Dynamical Systems, and Symmetry. P. Holmes, J. L. Lumley, and G. Berkooz, Cambridge University Press, New York, 1996, pp. 420.

This book is about the application of the theory of dynamical systems to turbulent flows. The present theory of dynamical systems pertains to low dimensions of the order 2, but turbulence is a high-dimensional phenomenon (of the order of 10^{18} in the surface layer of the atmosphere). What, then, is the overlap? The point is that significant reduction of dimensionality may be possible if one restricts attention to selected subdomains in physical and spectral spaces. The restriction necessarily excludes small scales of turbulent motion, and also large parts of some commonly studied flows; yet, where the dynamics is dominated by strong coherent structures, a useful approximation by a relatively small number of modes might be possible. As the authors of this book emphasize, this number may still be rather large from the perspective of dynamical systems, yet orders of magnitude smaller than the total number of degrees of freedom known to exist in practical turbulent flows. Occasionally, as in the case of the region very close to the wall—where coherent structures in the form of longitudinal vortices play a key role—the number of modes needed might be surprisingly small. This is the central perspective of the book.

To assess the value of this approach, one still needs to address the issue of how coherent structures should be described when the only access one has to be a turbulent flow (aside from the knowledge of the governing equations) is to some form of empirical data. The authors make use of the Proper Orthogonal Decomposition (POD)—better known, perhaps, to the readers of this Journal as the Karhunen-Loeve decomposition. The decomposition provides the most useful basis for the modal decomposition of empirical data and is optimal in the sense of capturing dominant components of a high-dimensional system by the smallest number of modes. One can then use a Galerkin projection of the governing equations on to

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the finite-dimensional subspace so constructed, and develop tractable low-dimensional dynamical models.

The book introduces these concepts and illustrates their application. It is aimed for use by engineers in aerospace, chemical, civil, environmental and geophysical areas, as well as to physicists and applied mathematicians. The diverse nature of the intended audience necessitates the inclusion of significant introductory material (about 60% of the book). The first two chapters introduce some well-known concepts and dimensional arguments found indispensable for the understanding of classical shear flows and coherent structures. The next two chapters introduce essential elements of the POD methodology and the use of Galerkin projection. The next four chapters contain basic material on dynamical systems, such as structural stability, local and global bifurcations, heteroclinic cycles, symmetry breaking, chaos and strange attractors, effect of noise on the evolution of dynamical systems, and so forth. The remaining four chapters bring together the multiplicity of these concepts through applications to specific examples. Two of these four chapters present a detailed account of the authors' own work (and that of their close collaborators) on the near-wall region of the boundary layer. The penultimate chapter is a summary of further applications to circular jets, forced mixing layers, and grooved channels. The concluding chapter is a personal account of the future prospects for the general approach advocated in the book.

It must be said that the book does not presume to develop a general theory of turbulence; rather, it seeks to develop an approach by means of which important dynamical processes in turbulence—such as the burst-sweep cycle of the boundary layer—can be extracted under appropriately favorable circumstances. In this regard, the book is a remarkable success. The exposition is scholarly, and the style lucid and lively. The book represents one of the refreshing developments in the modern era on turbulence research, and should be of value to everyone interested in turbulence. I recommend it wholeheartedly.

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